SPECTRAL SHAPING OF A RANDOMIZED PWM DC-DC CONVERTER USING MAXIMUM ENTROPY PROBABILITY DISTRIBUTIONS

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Abstract: The use of random pulse width modulation has been known to help reduce Electromagnetic Interference (EMI) in power converters. The procedure for reducing EMI using Random Pulse Width Modulation (RPWM) techniques has always been to spread the harmonic power in the power density spectrum -otherwise caused by periodic PWM- by randomizing the PWM. This in turn reduces the amplitude of the EMI noise. Probability distribution functions (PDF) determine the extent to which EMI amplitudes could be reduced. This paper presents probability distribtions of the PWM which will result in the spreading out of harmonic power in the power density spectrum. The relationship between probability distributions and spreading out of harmonic power whilst maintaining constraints in a DC-DC converter is investigated. A probability distribution whose aim is to ensure maximal harmonic spreading and yet mainaint constraints is presented. The PDFs are determined from a direct application of the method of Maximum Entropy. It is shown that given a pool of randomized parameters, there exists a region through which maximum spreading happens and how this spreading out is compromized by having constraints.

Keywords: Electromagnetic Interference, Maximum Entropy, Power Density Spectrum, Probability Distribution, RPWM

1. INTRODUCTION

In Power electronic converters, high speed semiconductors have been known to produce Electromagnetic Interference (EMI). The operation of periodic switching as well as large rates of change of currents and voltages are what produces the EMI. Part of this noise is conducted from within the switching circuit itself (conducted EMI) and part of it is radiated (radiated EMI) [1, 2]. Both of which are undesireable.

The main source of conducted EMI is the periodic switching operation in power converters, where the switching process is driven by a Pulse Width Modulation (PWM) signal. Periodic PWM signals follow a constant switching frequency (f_s) with a given duty ratio [3]. The spectrum results in large and 'sharp' amplitude peaks at the fundamental switching frequency and at multiples of this switching frequency.

Throughout the years these effects had been mitigated by using numerous varieties of carefully designed EMI filters. Inserting filters at the supply as well as at the output of the converters has been the primary means of attenuating conducted EMI [3]. However, this technique has not proved to be the best solution as it would result in increased volume and cost of power converters [2].

Since the mid 1980s there has been some research done on reducing the EMI without the need of EMI filters. It has been shown that it is possible to alter both common mode and differential mode conducted noise by altering the power spectral density (PSD) [2, 4]. Introducing some form of randomness to the PWM signal 'weakens' the periodicity of the PWM and consequently relaxes the harmonic power concentration at the frequencies [5]. Effects such as audible acoustic noise in motors arising from conducted EMI can be considerably reduced [6,7]. This addition of randomness to improve EMI performance is known as Dithering [2].

The reduction in EMI is largely dependent on the type of dithering technique. Every randomization strategy attempts to reduce the harmonic peaks as much as possible. However this must be done under some power conversion constraints such as keeping a constant average duty ratio. As such, a number of Random Pulse Width Modulation (RPWM) papers have been published for different types of power converters. The most fundamental type of RPWM is to randomize either the pulse width, pulse position or the period (switching frequency). Other "customized" techniques have also been developed to achieve a better performance than the fundamental ones [5, 6, 8, 9].

Since randomization is defined by the probability distribution of the random variables, then a probability distribution function (PDF) can be thought of as an input function to shaping the PSD. By altering this probability,

one is effectively able to alter the PSD [4, 5, 10–13].
The studies on using RPWM for altering the spectral content of a converter create an opportunity to 'custom shape' -to some extent- the output spectrum. The general approach to the spectral shaping problem is from an optimization perspective, where a PDF must be found that satisfies the following requirements: 1) must minimize a certain objective function or meet some spectral specifications, 2) must minimize this objective function within a certain region of constraints [4, 8, 14–17]. For example, this objective function can be the total power in a certain frequency band of the converter, and the constraints could be an average duty ratio that results in the desired power conversion (on average).
Generally, the technique of Random Modulation is chiefly concerned with EMI reduction. The work in this paper is also focused on the problem of spectral shaping for this purpose. The novelty lies in the nature of the 'objective function': to reduce the high amplitude peaks by spreading out the harmonic power in the frequency spectrum, as much as possible, while obeying the constraints.
This can be achieved through the use of the method of Maximum Entropy (MaxEnt). It is proposed to be the best suited optimization approach to finding probability distributions which will result in the specified requirements. As will be seen shortly, this approach further creates an opportutnity to establish the limitations of Random Modulation in EMI reduction.

2. PRINCIPAL OF RANDOM MODULATION

Many studies show that introducing randomness to the PWM alters the spectral content of the PWM. This impact varies depending on how this randomization is done.

2.1 Power Density Spectrum Spreading

In order to reduce the EMI, the harmonic power in the PSD must be reduced. Reducing the high amplitudes transfers the harmonic power to surrounding frequencies. An example of this phenomenon is seen in many research outputs such as [5, 6] to name a few. Therefore, spreading harmonic power is the fundamental aim in random modulation.

2.2 Impact of Random Modulation

A PWM can be characterised by three parameters as demonstrated by Figure 1. That is, pulse width W, the pulse position Δ and the period T.

Random Pulse Width (RPW) randomly varies W at every clock cycle. According to Tse *et al.* this randomization technique still contains discrete harmonics at multiples of the switching frequency with a continuous component over the frequency spectrum [5].
Random Pulse Position (RPP) modulation randomizes the pulse in the position located by the delay Δ within the

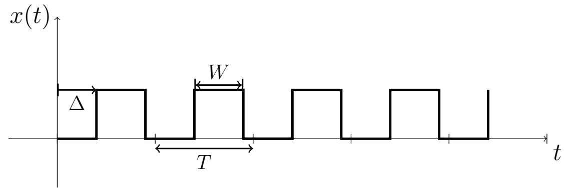


Figure 1: General PWM pulse train with three fundamental parameters

period T [5, 6]. The pulse position can be split into two other categories viz. lead and lag RPP [6, 8]. According to the authors in [6], randomizing this variable results in each harmonic being reduced accordingly and the power lost is transferred to the continuous spectrum. However the harmonics still have relatively high amplitudes but are progressively smaller for higher order harmonics.
If the period T is randomized, while keeping the pulse width W constant then the duty cycle changes accordingly. This is referred to as random carrier frequency with variable duty modulation (RCFVD), else if the width is allowed to vary by the same proportion as the period, then it becomes a random carrier frequency with fixed duty modulation (RCFFDM) [5].
The PSD of the RCFVDM and RCFFDM has a continuous spectrum due to the switching frequency itself being randomized. According to [6] randomization of the switching frequency flattens the discrete harmonics into the continuous spectrum. But the low frequency harmonics in RCFVDM has higher amplitidues than that of RCFFDM [5]. This makes the RCFFDM to be the best among the three fundamental random modulation techniques when it comes to 'spreading' harmonic power [5].

Customized random modulation techniques can be seen as a hybrid combination of any of the three degrees of freedom Δ, W and T [9]. Other customized techniques aim to control the randomization strategy by implementing customized algorithms that generate controlled noise through which the switching pattern is derived. [9, 18–20]. Most of these strategies are based on uniform probability distributions, which are sufficient for randomization, but without guaranteeing as to whether these are the best suited probability distributions for shaping or spreading the PSD.

2.3 Spectral Shaping

The idea behind spectral shaping is to select a randomization technique with its associated PDF to analytically obtain a specified spectral profile [21]. The benefits of this idea comes in being able to achieve some level of controllability on the spectral content. To achieve this, a model that analytically links the time domain PDF of the random variables and the frequency domain would be required. From there an optimization strategy can be employed to approximate a prescribed spectral profile. Bech's research proposed an approximation model that provides this link [4]. It generalizes the random modulation PDFs of each random variable in the PWM and calculates the spectral content. However, the problem of finding an ideal PDF still remains a challenge as there are many possible PDF families to choose from. However, using discrete PDFs with a selected pool for each of the RPWM parameters helps mitigate this problem [4,22]. The associated PDF weightings become the design variables found through some optimization strategy for the spectral requirements to be achieved [4]. This is the approach used here.

3. MAXIMUM ENTROPY PROBABILITY DISTRIBUTIONS

The harmonic power in the PSD must be spread out in order to reduce the high amplitudes. Therefore, one would ideally want to maximally spread the harmonic power by altering PDFs. If this notion of 'spreading' could be quantified, then one would have a quantifiable performance index through which an optimization strategy can be employed. The method of MaxEnt makes use of this criteria while maintaining whatever constraints are imposed.

3.1 Maximal Harmonic Spreading

It has been established that an ideal PSD is one that has a maximally spread harmonic power. This implies that the harmonic peaks would be at their lowest which in turn implies less EMI. If the PSD can be altered by altering the probability distribution of the random variables, then there is reason to believe that a spreading out of probability leads to a spreading out in the PSD. For example, a uniform probability distribution of the carrier frequency aims at uniformly spreading out frequencies in the PSD [17]. The next step would then be to impose power conversion constraints and maximally spread out the probability so that the PSD can be maximally spread out.

3.2 Maximum Entropy

Spreading out of probability can be quantified by calculating the Shannon's entropy $S[P]$ of the discrete probabilities as shown in equation (1) [23].

$$S[P] = -\sum_{i=1}^{n} P_i \log P_i \tag{1}$$

Now MaxEnt is a tool that allows one to assign probabilities that maximize entropy (or probability spreading) and agrees with whatever constraints imposed [23]. The goal is to select the distributions P_i from maximum entropy, for which the expectation (or constraints) of some functions $f^k(x)$,(where $k = 1, 2, \dots$) is known to have the numerical values F^k, i.e. P_i must satisfy equation (2) below.

$$\mathbf{E}\Big[f^k(x)\Big] = \sum_i P_i f^k(x_i) = F^k \tag{2}$$

Where $\mathbf{E}[\cdot]$ is the Expectation operator. The entropy maximization is achieved by setting:

$$0 = \delta\Big(S[P] - \alpha \sum_i P_i - \lambda_k \mathbf{E}\Big[f^k(x)\Big]\Big) \tag{3a}$$

$$= -\sum_i \Big(\log P_i + 1 + \alpha + \lambda_k f^k(x_i)\Big)\delta P_i \tag{3b}$$

Where δ here represents the first variation, α and λ_k are Lagrange multipliers [23].
Solving equation (3) yields what is called the Canonical probability distribution [23]:

$$P_i = e^{-(\lambda_0 + \lambda_1 f_i^k)} \tag{4}$$

Where $\lambda_0 = 1 + \alpha$ is determined from the normalization constraint $\sum_i P_i = 1$, and λ_k is determined from the constraints in equation (2). It must be noted therefore that entropy is maximized within the region of constraints, if the region of constraints is 'enlarged' so to speak, then the maximized entropy will be larger than in the smaller region.

4. MAXIMUM ENTROPY PDF FOR RPWM

In the case of Random Modulation, there are three parameters which are subject to probability distributions. In general, the pulse width, pulse period and position have the pool:

$$T_i = \{T_1, T_2, ..., T_{N_T}\} \tag{5a}$$

$$\Delta_j = \{\Delta_1, \Delta_2, ..., \Delta_{N_j}\} \tag{5b}$$

$$W_l = \{w_1, w_2, ..., w_{N_W}\} \tag{5c}$$

And normalization constraints:

$$\sum_{i}^{N_T} P_{T_i|\Delta,W} = 1 \tag{6a}$$

$$\sum_{j}^{N_\Delta} P_{\Delta_j|W,T} = 1 \tag{6b}$$

$$\sum_{l}^{N_W} P_{W_l|\Delta,T} = 1 \tag{6c}$$

Where $P_{T_i|\Delta,W}$, $P_{\Delta_j|W,T}$ and $P_{W_l|\Delta,T}$ are the conditional stationary probabilities given the remaining parameters. And $T > 0$, $0 \leq W \leq T$ and $T_i \leq \Delta \leq T_{i-1}$, which ensures that this remains a valid PWM.
MaxEnt will then be applied in order to find the probability distribution within three different sets of constraints: probability distributions in maximal constraints, probability distributions in minimal constraints and a set of constrains which lie in between. In each case, the resulting PWM and its PSD are investigated. The distribution here is solved for the pulse period $P_{T_i|\Delta,W}$ only, however the same approach can be used for the remaining parameters.

4.1 PDF with Maximum Constraints

For maximum constraints, the pool for the parameters is limited to only one possible outcome.

$$T_i = \{T_1\} \tag{7}$$

With the following constraint

$$\mathbf{E}[T] = \sum_{i}^{N_T} P_{T_i|\Delta,W} T_i = T_1 \tag{8}$$

The canonical distribution for T is therefore:

$$P_{T_i|\Delta,W} = e^{-(\lambda_0+\lambda_1 T_i)} \tag{9}$$

From the normalization constraint, the equation below is obtained:

$$\sum_{i}^{N_T} e^{-\lambda_1 T_i} = e^{(\lambda_0)} \tag{10}$$

since $N_T = 1$, the distribution then becomes:

$$P_{T_i|\Delta,W} = e^{-(-\lambda_1 T_1+\lambda_1 T_1)} \tag{11}$$

$$\therefore P_{T_i|\Delta,W} = \delta(T_i - T_1) \tag{12}$$

Here, δ is the Kroneker delta function. So according to MaxEnt, this is the 'most spread out' distribution that meets all constraints. However, due to these constraints, the probability is certain and therefore non-random. The constraints limit the extent to which T can be randomized. The PWM would therefore result in the worst possible PSD as there is no randomization. The ideal PSD would be one with the most randomization, meaning minimal constraints.

4.2 PDF with Minimum Constraints

Minimum constraints means that there are many degrees of freedom for randomization. Thus the pool's size can be of any possible size $N_T > 1$. There are no constraints, except for the normalization constraints in equation (6a). Therefore only the multiplier λ_0, remains so that the Canonical distribution becomes

$$P_{T_i|\Delta,W} = e^{-\lambda_0} \tag{13}$$

From the normalization constraint, the distribution is found as:

$$P_{T_i|\Delta,W} = 1/N_T \tag{14}$$

Which is a uniform distribution accross the entire pool. That is, the most spread out or MaxEnt PDF without constraints is a uniform distribution.

4.3 PDF with Some Constraints

Constraints that lie between the two previous cases are now considered. This case is better illustrated with a numerical example.

Say the pool for the carrier period T is given:

$$T_i = \{T_1, T_2, ..., T_{N_T}\} \tag{15}$$

With the following constraints:

$$\mathbf{E}[T] = \sum_{i}^{N_T} P_{T_i|\Delta,W} T_i = 2W \tag{16}$$

The normalization constraint determines λ_0, so that $P_{T_i|\Delta,W}$ becomes:

$$P_{T_i|\Delta,W} = \frac{e^{-\lambda_1 T_i}}{\sum_i^{N_T} e^{-\lambda_1 T_i}} \tag{17}$$

From the constraint in (16):

$$\sum_{i}^{N_T} P_{T_i|\Delta,W} T_i = \frac{\sum_i^{N_T} T_i e^{-\lambda_1 T_i}}{\sum_i^{N_T} e^{-\lambda_1 T_i}} = 2W \tag{18}$$

Solving the above equation is non-trivial, but can be solved numerically. Let the pool of carrier periods be $T_i = \{10s, 15s, 20s, 25s, 30s\}$. If the right hand side of (18) is set to be equal to some variable y, then a plot of y versus λ_1 can be obtained and is shown in Figure 2. This is a plot of all possible Expectations (or constraints) with the available pool versus the Lagrange multiplier λ_1. Since it is required that $y = 2W$, and say $W = 12,5s$, then the dotten line in Figure 2 marks the point where $y = 25s$ and the corresponding $\lambda_1 = -0.01123$ at that point.
From this, the distribution for the carrier period is therefore:

$$P_{T_i|\Delta,W} = \frac{e^{0.01123 T_i}}{\sum_i e^{0.01123 T_i}} \tag{19}$$

And its shape is plotted in Figure 3.

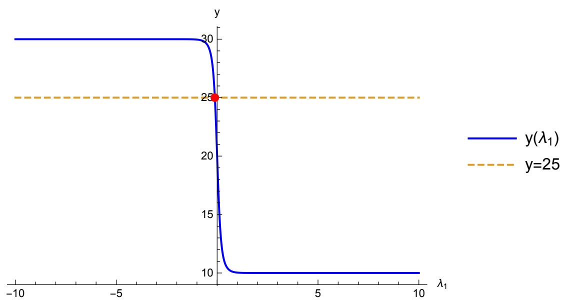


Figure 2: Plot showing available averages for the constraints y with associated Lagrange multiplier λ_1

This is the MaxEnt distribution of the carrier period that maximises spreading yet obeys constraints.

4.4 MaxEnt PDF and Associated PSD

The MaxEnt distribution is now applied to an example of a 100Hz PWM for a DC-DC converter switch with a

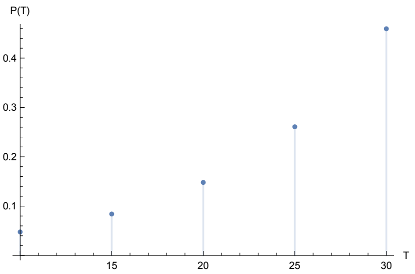
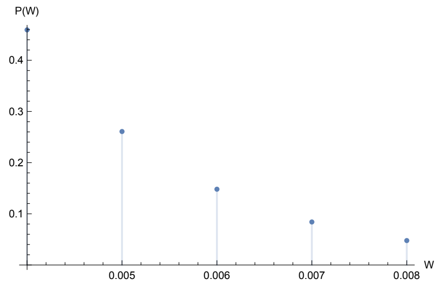
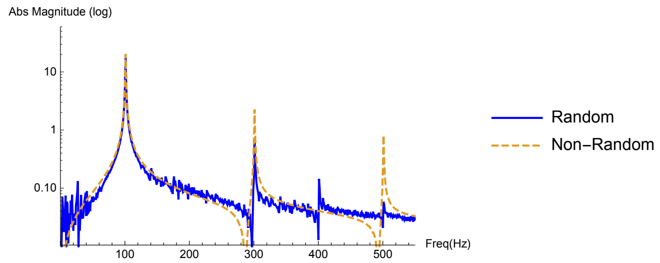


Figure 3: MaxEnt Probability Distribution for carrier period T with average period $\mathbf{E}[T] = 25s$



(a) Maximum entropy probability distribution for pulse width W



(b) PSD for random (W) with **some constraints** in the given pool, versus non-random pulse width.

Figure 4: MaxEnt probability distribution of Random pulse width (W) (a) and its associated PSD (b).

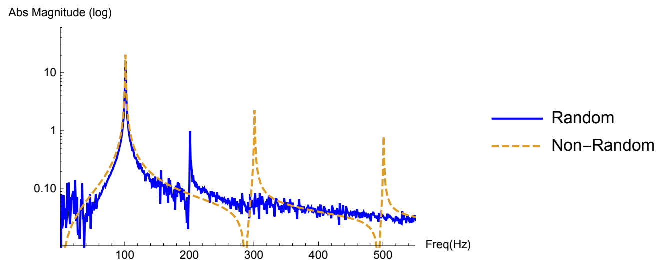


Figure 5: Comparison of PSD for random pulse width (W) with **no constraints** in the given pool (from uniform MaxEnt distribution), versus non-random pulse width.

50% duty cycle. The resulting PSD is observed. For this application the pulse width W is used as the random variable with the following pool.

$$W_l = \{1ms, 2ms, 2.5ms, 5ms, 7.5ms\} \tag{20}$$

The constraints are that the average pulse width must be half the pulse period to ensure a duty cycle of 50%-on average:

$$\mathbf{E}\{W\} = \sum_{l}^{N_W} P_{W_l|T,\Delta} W_l = T/2 = 5ms \tag{21}$$

The MaxEnt probability distribution found is:

$$P_{W_l|T,\Delta} = \frac{e^{-566.1W_l}}{\sum_l^{N_W} e^{-566.1W_l}} \tag{22}$$

Which is plotted in Figure 4a. The associated PSD is obtained by taking the Fourier transform of the Autocorrelation of the RPWM signal [4]. This is plotted in Figure 4b. The PSD can be compared to that of a deterministic or non random PWM in dashed lines where the harmonics have higher amplitudes. Clearly by randomizing the pulse width, the amplitudes have been reduced and their power spread to other frequencies. This is the most spread out PSD possible as obtained from MaxEnt PDF, given the pool with the provided converter constraints. However, if the constraint in (21) need not be obeyed, then the most spread PSD with no constraints can be obtained. This is plotted in Figure 5. As shown by the absolute magnitides of the harmonics, it is also clear that the harmonic power has been better spread out than in Figure 4b where there are some constraints.

5. DISCUSSION

From what has been presented thus far, it is safe to state that; by considering the maximum and the minimum possible constraints, one is effectively able to define a region in which randomization can be done. In this region, MaxEnt provides the most spread out distribution for whatever set of constraints that lie in the region. As a result one is able to prioritise harmonic power spread in PSD while obeying constraints. However, there exists a trade-off between constraints and PSD spreading. Essentially, constraints can be seen as the limiting factors to PSD spreading and to EMI reduction.

The MaxEnt probability distribution presented has however not been generalized for a general PWM. For this, one would have to consider the joint probability distribution of W, Δ and T, instead of the conditional distributions. From this, the relationship between all possible constraints, probability distributions and the PSD spectrum can be generalized for the purpose of spectral shaping.

6. CONCLUSION

An approach for determining the PDF of a RPWM that aims to maximally spread the harmonic power in the

PSD while obeying constraints has been presented. This appoach is based on the direct application of the method of MaxEnt. This method allowed for an investigation into the relationship between the constraints, spreading in the PSD and the PDFs. It has therefore been shown that by considering maximum and minimum possible constraints, a region in which maximal spreading will occure can be defined. And by imposing constraints on the PDF, one effectively moves away from obtaining the most (or ideal) PSD spreading that can possibly be obtained. The investigated relationship, must however, still be generalized.

REFERENCES

[1] F. Mihali and D. Kos. "Reduced Conductive EMI in Switched-Mode DC ndash;DC Power Converters Without EMI Filters: PWM Versus Randomized PWM." *IEEE Transactions on Power Electronics*, vol. 21, no. 6, pp. 1783–1794, Nov. 2006.

[2] A. Elrayyah, K. M. Namburi, Y. Sozer, and I. Husain. "An Effective Dithering Method for Electromagnetic Interference (EMI) Reduction in Single-Phase DC/AC Inverters." *IEEE Transactions on Power Electronics*, vol. 29, no. 6, pp. 2798–2806, Jun. 2014.

[3] D. Gonzalez, J. Balcells, A. Santolaria, J. C. L. Bunetel, J. Gago, D. Magnon, and S. Brehaut. "Conducted EMI Reduction in Power Converters by Means of Periodic Switching Frequency Modulation." *IEEE Transactions on Power Electronics*, vol. 22, no. 6, pp. 2271–2281, Nov. 2007.

[4] M. M. Bech, F. Blaabjerg, and J. K. Pedersen. "Random modulation techniques with fixed switching frequency for three-phase power converters." *IEEE Transactions on Power Electronics*, vol. 15, no. 4, pp. 753–761, Jul. 2000.

[5] K. K. Tse, H. S. H. Chung, S. Y. R. Hui, and H. C. So. "A comparative study of using random switching schemes for DC/DC converters." In *Applied Power Electronics Conference and Exposition, 1999. APEC '99. Fourteenth Annual*, vol. 1, pp. 160–166 vol.1. Mar. 1999.

[6] A. M. Trzynadlowski, F. Blaabjerg, J. K. Pedersen, R. L. Kirlin, and S. Legowski. "Random pulse width modulation techniques for converter-fed drive systems-a review." *IEEE Transactions on Industry Applications*, vol. 30, no. 5, pp. 1166–1175, Sep. 1994.

[7] G. Wang, L. Yang, B. Yuan, B. Wang, G. Zhang, and D. Xu. "Pseudo-Random High-Frequency Square-Wave Voltage Injection Based Sensorless Control of IPMSM Drives for Audible Noise Reduction." *IEEE Transactions on Industrial Electronics*, vol. 63, no. 12, pp. 7423–7433, Dec. 2016.

[8] R. L. Kirlin, S. Kwok, S. Legowski, and A. M. Trzynadlowski. "Power spectra of a PWM inverter with randomized pulse position." In *, 24th Annual IEEE Power Electronics Specialists Conference, 1993. PESC '93 Record*, pp. 1041–1047. Jun. 1993.

[9] K. S. Kim, Y. G. Jung, and Y. C. Lim. "A New Hybrid Random PWM Scheme." *IEEE Transactions on Power Electronics*, vol. 24, no. 1, pp. 192–200, Jan. 2009.

[10] M. M. Bech, J. K. Pedersen, F. Blaabjerg, and A. M. Trzynadlowski. "A methodology for true comparison of analytical and measured frequency domain spectra in random PWM converters." *IEEE Transactions on Power Electronics*, vol. 14, no. 3, pp. 578–586, May 1999.

[11] R. L. Kirlin, M. M. Bech, and A. M. Trzynadlowski. "Analysis of power and power spectral density in PWM inverters with randomized switching frequency." *IEEE Transactions on Industrial Electronics*, vol. 49, no. 2, pp. 486–499, Apr. 2002.

[12] R. L. Kirlin, S. F. Legowski, and A. M. Trzynadlowski. "An optimal approach to random pulse width modulation in power inverters." In *, 26th Annual IEEE Power Electronics Specialists Conference, 1995. PESC '95 Record*, vol. 1, pp. 313–318 vol.1. Jun. 1995.

[13] R. L. Kirlin, C. Lascu, and A. M. Trzynadlowski. "Shaping the Noise Spectrum in Power Electronic Converters." *IEEE Transactions on Industrial Electronics*, vol. 58, no. 7, pp. 2780–2788, Jul. 2011.

[14] A. M. Stankovic, G. E. Verghese, and D. J. Perreault. "Analysis and synthesis of randomized modulation schemes for power converters." *IEEE Transactions on Power Electronics*, vol. 10, no. 6, pp. 680–693, Nov. 1995.

[15] T. S. Kiran and K. N. Pavithran. "Novel approach for harmonic reduction with random PWM technique for multilevel inerters." In *2015 International Conference on Power, Instrumentation, Control and Computing (PICC)*, pp. 1–6. Dec. 2015.

[16] A. M. Trzynadlowski, M. M. Bech, F. Blaabjerg, J. K. Pedersen, R. L. Kirlin, and M. Zigliotto. "Optimization of switching frequencies in the limited-pool random space vector PWM strategy for inverter-fed drives." *IEEE Transactions on Power Electronics*, vol. 16, no. 6, pp. 852–857, Nov. 2001.

[17] R. L. Kirlin and A. M. Trzynadlowski. "Spectral Design of Randomized Pulse Width Modulation in DC to AC Converters." In *, IEEE Seventh SP Workshop on Statistical Signal and Array Processing*, pp. 387–391. Jun. 1994.

[18] T. Tanaka, H. Hamasaki, and H. Yoshida. "Random-switching control in DC-to-DC converters: an implementation using M-sequence." In *Telecommunications Energy Conference, 1997. INEC 97., 19th International*, pp. 431–437. Oct. 1997.

[19] H. Li, Y. Liu, J. L, T. Zheng, and X. Yu. "Suppressing EMI in Power Converters via Chaotic SPWM Control Based on Spectrum Analysis Approach." *IEEE Transactions on Industrial Electronics*, vol. 61, no. 11, pp. 6128–6137, Nov. 2014.

[20] L. Premalatha and A. Thanuja. "Generation of chaos and EMI reduction in current controlled boost converter using Random modulation." In *2011 International Conference on Recent Advancements in Electrical, Electronics and Control Engineering (ICONRAEeCE)*, pp. 211–215. Dec. 2011.

[21] A. Carlosena, W.-Y. Chu, B. Bakkaloglu, and S. Kiaei. "Randomized carrier PWM with exponential frequency mapping." In *2006 IEEE International Symposium on Circuits and Systems*, pp. 4 pp.–. May 2006.

[22] A. M. Trzynadlowski, M. M. Bech, F. Blaabjerg, J. K. Pedersen, R. L. Kirlin, and M. Zigliotto. "Optimization of switching frequencies in the limited-pool random space vector PWM strategy for inverter-fed drives." *IEEE Transactions on Power Electronics*, vol. 16, no. 6, pp. 852–857, Nov. 2001.

[23] A. Caticha. "Lectures on Probability, Entropy, and Statistical Physics." *arXiv:0808.0012 [cond-mat, physics:physics, stat]*, Jul. 2008. URL `http://arxiv.org/abs/0808.0012`. ArXiv: 0808.0012.