An effective and efficient method of calculating Bessel beam fields

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ABSTRACT

Bessel beams have gathered much interest of late due to their properties of near diffraction free propagation and self reconstruction after obstacles. Such laser beams have already found applications in fields such as optical tweezers and as pump beams for SRS applications. However, to model the self reconstruction property of Bessel beams, it is necessary to calculate the field at all points in space before and after the obstacle – a computationally intensive task give the large spatial distribution of Bessel beams. In this work we propose a computationally efficient method of calculating the arbitrary propagation of a Bessel beam, which is both fast and accurate. This method is based on transforming the problem to a new co-ordinate system more in line with the conical nature of the wavefronts, and shows excellent agreement with more traditional methods of calculation based on the Kirchoff-Fresnel diffraction theory in cylindrical co-ordinates. The success of the method is shown for the case of Bessel beams and Bessel-Gauss fields passing through non-transparent obstacles, as well as the for case of these fields propagating through a scattering medium.

Keywords: Bessel beams, scattering media, conical wavefronts, diffraction, beam propagation

1. INTRODUCTION

Bessel light beams (BLBs) and their properties have been extensively studied, and are well documented in the literature[1-8]. One of the important properties of BLBs is the reconstruction of their amplitude and phase immediately behind an obstacle[9-15]. This property has been exploited for a range of applications, from optical tweezers to the optical probing of scattering media. The property can be explained by considering the reconstruction of the transverse amplitude profile behind obstacles[10], and has been confirmed experimentally for arbitrarily shaped obstacles[10]. Various groups have considered this self reconstruction property under different conditions, including in a nonlinear medium[11], by wave packets due to spatial – temporal links[12], and due to 2D non-periodical objects[13]. The quality and accuracy of periodical self reconstruction was investigated[14], as was the propagation of BLBs through media containing scattering centers[15]. Despite this large body of work, the calculation methods have not progressed to a stage where they are fast and accurate, and capable of calculating the BLB field as it passes through arbitrary objects.

In order to efficiently use the reconstruction property mention above, a method is required that allows for the obstructed field to be calculate at any point, both quickly and accurately. The standard method to retrieve the field is based on the Kirchoff-Fresnel diffraction theory, and is computationally intensive due to the time required to solve the characteristic double integral over a large transverse. One can reason that the source of the problem is in fact not the diffraction equation itself, but rather the co-ordinate system used in the description of problem. It is customary to solve the diffraction equation in cylindrical coordinates, which breaks the symmetry of the conical wavefronts describing a set of BLBs.

In the remaining sections we first introduce a new conical coordinate system, and show how transformations can be made from the cylindrical to the conical system and back. We then show how this transformation aids the fast and accurate calculation of BLBs behind arbitrary obstacles, and outline some examples where diffraction effects are include, and so where they are not.

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2. CONICAL COORDINATE SYSTEM

As mentioned in the previous section, the use of a cylindrical coordinate system is not ideal when describing fields with conical symmetry, which is the case for BLBs. Another limitation in the speed and accuracy of calculations is the fact that traditionally a full account of all the diffraction effects has been required for the calculations.

We could therefore expect an appreciable improvement in operating speed, while still retaining adequate accuracy, if it were possible to use geometric approximations in a coordinate system compatible with the conical symmetry of the BLB wavefronts.

To do this we must find a coordinate system that conforms to the symmetry of the spatial wavevectors (normal to the wavefronts) of the BLB, namely:

$$E = J_0(k_0 \sin(\gamma r) \exp(ik_z z) , \quad (1)$$

where γ is the cone angle, usually determined by the axicon in generating the BLB.

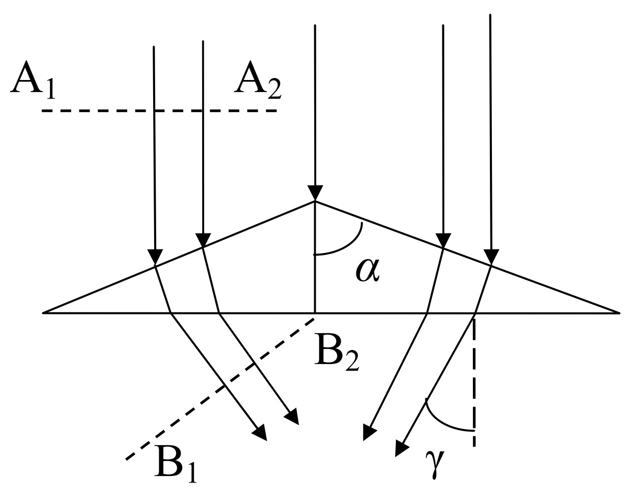

Figure 1a: Propagation of plate wave through axicon.

To fully appreciate the new coordinate system, it is worth revisiting the method of Bessel beam generation. A Bessel beam distribution is created by passing incident plane waves through an axicon (see figure 1a). On exiting the axicon, the wave vectors that were parallel follow the surface of a cone (i.e., always intersecting). In this case the wave front is a cone with angle γ (see figure 1 a). A value of this angle depends on both axicon angle α and index of refraction of the axicon. If one considers a plane of constant phase prior to the axicon ($A_1 - A_2$), and maps this to the new plane of constant phase after the axicon, then one sees that the plane is rotated by an angle γ, to form the new plane $B_1 - B_2$. The intersection of these angled plane waves is what generates the Bessel beam. Thus a Bessel beam in cylindrical coordinates is equivalent to a summation over many plane waves with wavefronts forming cones.

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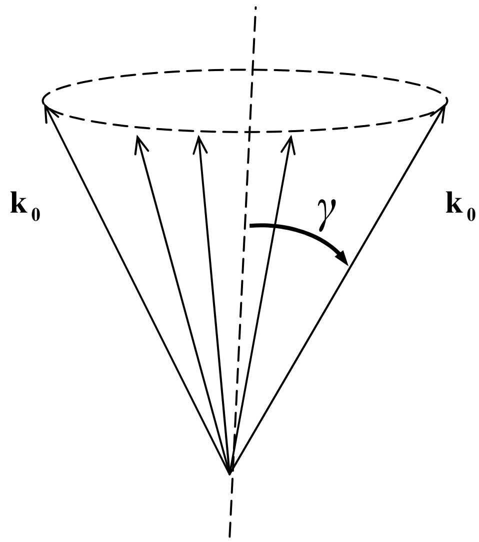


Figure 1b: The wave vectors (in bold) of the BLB form a cone of angle γ.

Consider an arbitrary point *A* in the Cartesian system, with the *z* axis parallel to the propagation direction of the BLB. Let's direct one of the axes (axis l in figure 2) in a new coordinate system transverse to the direction of the wave vectors of the BLB. Furthermore, let's choose the rest of axes in such a way that if this new axis is set to an angle of $\xi=\pi/2$, then the new coordinate system reverts to the conventional cylindrical (see figure 2).

As we see from figure 2, $\gamma = \pi/2 - \xi$, and when $\zeta = \pi/2$ the angle γ is zero. This is analogous to the BLB field distribution looking identical to a plane wave, since the wave vectors are all parallel. Thus, the cylindrical coordinate system can be considered as a special case of the conical coordinate system, and furthermore, the Bessel distribution per radial coordinate in the cylindrical system is similar to the field distribution of a plane wave in the conical system:

$$E = E_0 Exp(i\omega t + ik_0 z), \tag{2}$$

for the reasons discussed earlier.

Similarly, in the conical coordinate system the BLB propagation in homogeneous or inhomogeneous media, is similar to that of a plane wave propagating in a Cartesian coordinate system. The implications of this are as follows: in order to determine the BLB distribution at any position, one merely has to add the contributions from the relevant plane waves in the conical system. Since the addition of planes waves is easy and fast, the problem can be solved quickly, and then simply transformed back to the relevant coordinate system for the solution. Ordinarily this problem would be solved by staying in the cylindrical coordinate system and propagating the field through the obstacle using standard diffraction code.

In the sections that follow we illustrate the power of this method by considering the propagation of Bessel type fields through obstacles.

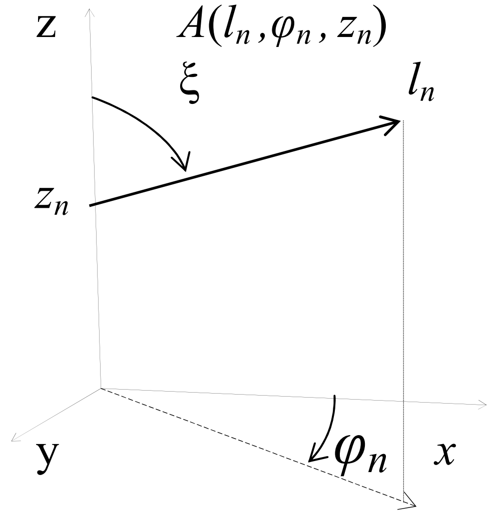


Figure 2: The conical coordinate system, with the new axis tilted to be consistent with the wave vectors of the BLB. When the angle between the new axis and the old z axis is ζ =π/2, we are again describing a cylindrical system. In the new system, the position of any point in space is given by the three coordinates (l, φ, z).

3. APPLICATION TO RECONSTRUCTION OF BESSEL BEAMS

In order to illustrate applying this new coordinate system to problems, consider the problem of finding the BLB field at some distance *z* (see dashed line in figure 3b) when a non-transparent obstacle in the form of sector ABCD lies on the path of the BLB. This is a reconstruction problem, which would usually be solved by taking into account the full effects of diffraction.

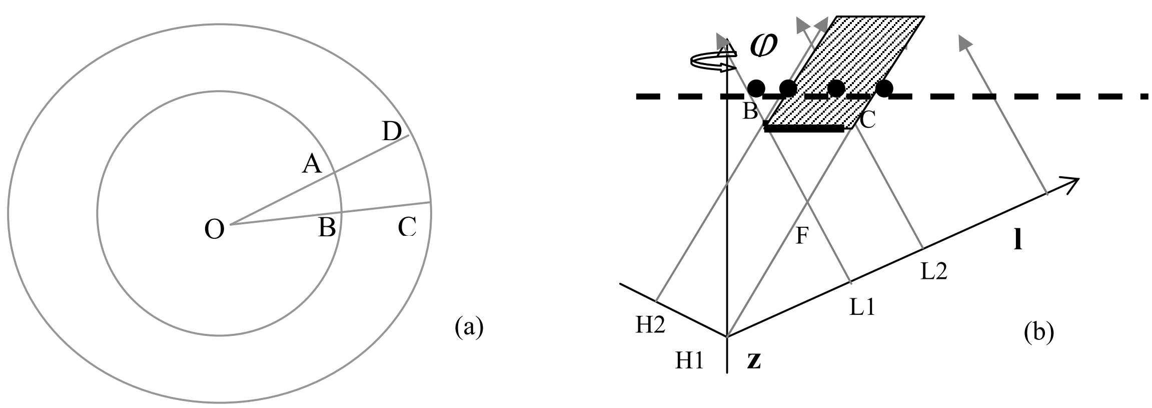
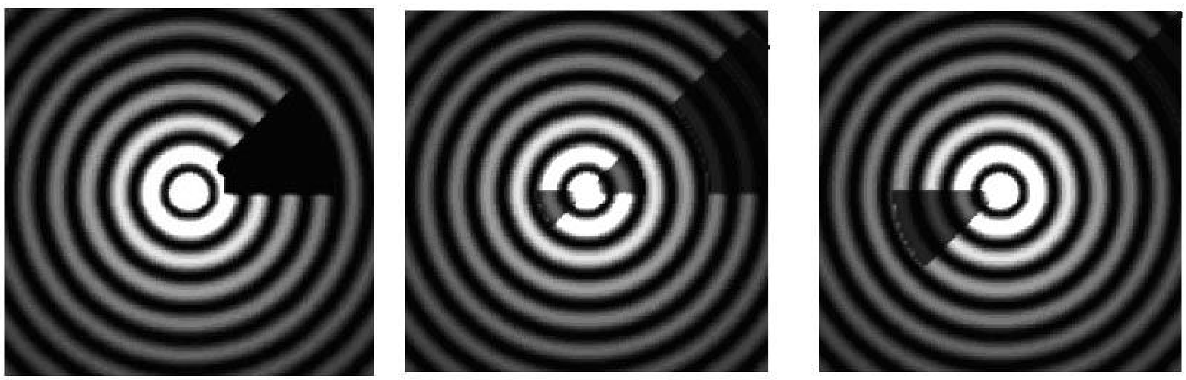


Figure 3: Geometric method of finding the field behind an obstacle in the conical coordinate system. The bold terms in (b) are vectors.

As we see from figure 3b, to find the BLB field after the obstacle ABCD (shown in figure 3a) we must consider the field behind the obstacle as if it were illuminated by a plane wave, i.e., by considering its geometric shadow. However, the

BLB has two intersecting wavefronts, which are shown as rays FC and FB (see figure 3b). The geometric shadow then becomes the area BC, which in the conical coordinate system is equivalent to extensions of regions $L_1 - L_2$ and $H_1 - H_2$ passing through the points B and C. This is illustrated in figure 3b for the $H_1 - H_2$ wave and is shown by a shaded region beyond B – C. Next, one must add the contribution from each wave long the observation plane (dashed line). Clearly there will be regions where only one plane wave contributes, where both plane waves contribute, and where no plane waves contribute. These transition points are shown in the figure as solid disks along the observation plane. Once the contribution from each wave is known everywhere along the observation plane, it suffices to convert back to the cylindrical coordinate system to visualize the result.

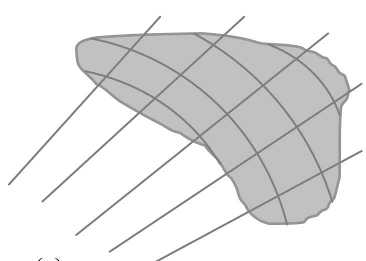
After applying this method to the above problem, we were able to rapidly and exactly calculate the evolution of the BLB field in the presence this obstacle. Figure 4a – 4c shows an example with a 532nm BLB with cone angle γ = 3.2 mrad passing over an obstacle of height 30μm.



(a) (b) (c)

Figure 4: The BLB passing over the obstacle (sector angle of $\alpha = \pi/4$). The distance to the obstacle l is 0.mm (a), 0.7 mm (b), and 1.2 mm (c).

More complicated obstacles can be handled in one of two ways: (a) One first dissects the obstacle into a set of sectors (see figure 5a), with the number of data points used a function of the required accuracy. The method described above can then be used to calculate the BLB behind each sector, or (b) A faster method is to mark on the obstacle some typical points, say A, B, C, and D (see figure 5b), and then to map their influence during the propagation of the BLB. In either case, we can reproduce the full structure of the BLB behind complicate obstacles by knowing the behavior of some typical points.



(a)

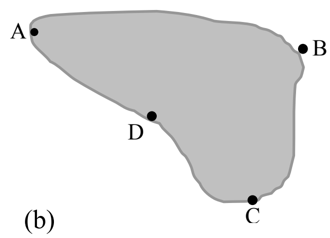


Figure 5: Arbitrary obstacles in the path of the BLB.

4. APPLICATION TO RECONSTRUCTION OF BESSEL – GAUSS BEAMS

Having shown that the coordinate system change correctly maps the reconstruction of Bessel beams, we now consider the case of modeling Bessel – Gauss light beams (BGLBs). As before, the first step is to convert the BGLB from the usual cylindrical coordinate system to the more computationally friendly conical coordinate system.

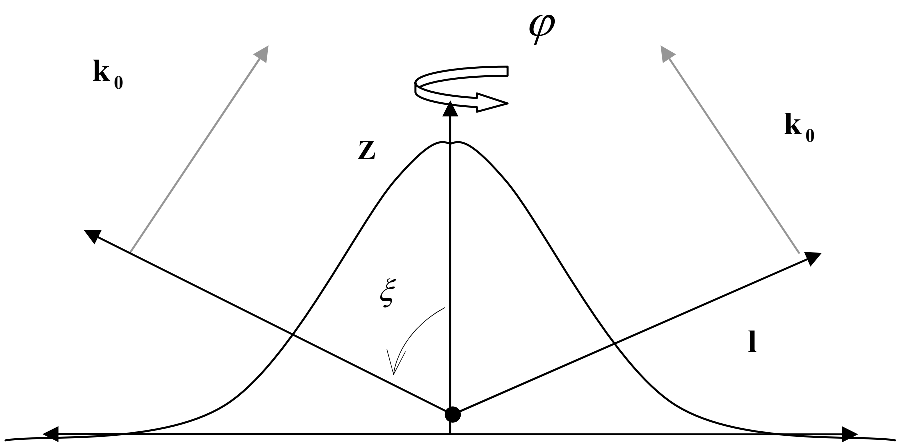
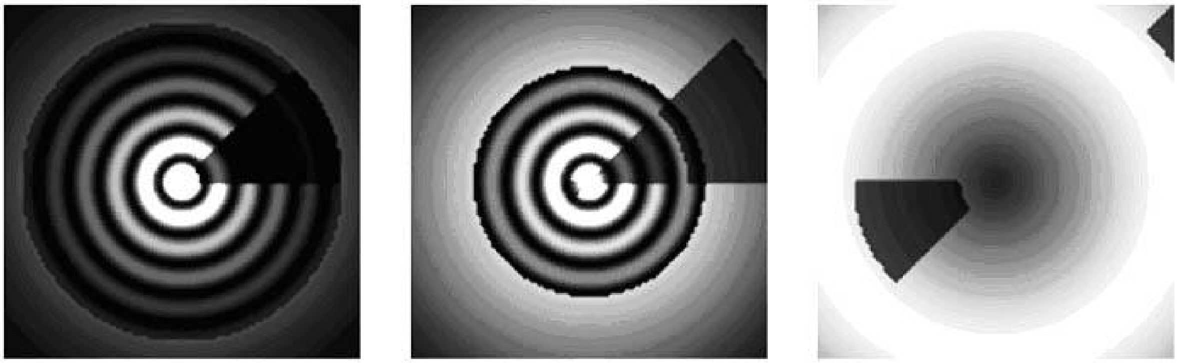


Figure 6: Visual demonstration of conversion of Gauss distribution from cylindrical CS to conical CS.

From figure 6 it is obvious that the BGLB in the conical system will again be described by a Gaussian envelope, but with a half width that is dependent on the transformation from the r axis in cylindrical coordinates to the l axis in conical coordinates. However, the field appears in the form of cylindrical waves in the conical system.

Figure 7 shows the results of modeling the BGLB after passing through an obstacle (same obstacle as in previous section). As we expect, the influence of obstacle in this case is the same as was considered for the BLB. The difference lies in the periphery area, where ring structure is absent.



(a) (b) (c)

Figure 7: The BGLB passes over the obstacle, with l = 0.1mm (a), 0.5 mm (b), 1.3 mm (c).

5. RECONSTRUCTION IN A SCATTERING MEIDA

In this section we consider the propagation of a BLB through a medium containing particles for which we can't neglect diffraction effects (recall in the previous sections that we considered only a geometric argument). To simplify the task, we make the following assumptions:

Firstly, that the particles can be treated as sectors similar to that of the previous sections (see figure 3a). Hence we can take AB = DC, and consequently we can consider square particles by modeling a square sector. Since by definition the particles are small, we assume that the wavefronts do not change direction while propagating inside the particle. The result is that we can treat the diffraction of the field inside the particles as if the field was a plane wave.

Secondly, that the typical distances between particles is large compared to the particle sizes themselves, and so we can use the Fraunhofer approximation to the diffraction problem.

By the first and second assumption, as well as Babine's principle[16], we can consider diffraction field picture from each particle in the form:

$$A(1-\sin(k_0 r_i r_0 / z)/(k_0 r_i r_0 / z)), \tag{3}$$

where A is the amplitude falling on particles, r_0 the particle radius, z the distance from particle to transparence, $r_i = r_p - r$, with r_p the particle coordinate. We must take into account the fact that the amplitude A is changed if we take into account the influence of other particles. The general equation for finding the amplitudes in such a medium is give by:

$$A = A_0 - \sum_{i=0}^{n} A_i \frac{\sin[x_{in}]}{x_{in}}, \tag{4}$$

with A_i the amplitude of the field on i^{th} particle, $x_{in} = k_0 r_i r_0 / z_n$, and z_n is the distance from i^{th} particle to the plane of interest.

In order to find the field in the plane of interest, we have to know the amplitudes of the field falling on each particle. We can find this from the following system of equations:

$$
\begin{cases}
A_1 = A_0 - A_0 \dfrac{Sin[x_{01}]}{x_{01}} \\
A_2 = A_0 - A_0 \dfrac{Sin[x_{02}]}{x_{02}} - A_1 \dfrac{Sin[x_{12}]}{x_{12}} \\
\text{K} \\
A_n = A_0 - \sum_{i=0}^{n-1} A_i \dfrac{Sin[x_{in}]}{x_{in}}
\end{cases}, \quad (5)
$$

where x_{in} is the distance from the i^{th} particle to the n^{th} particle.

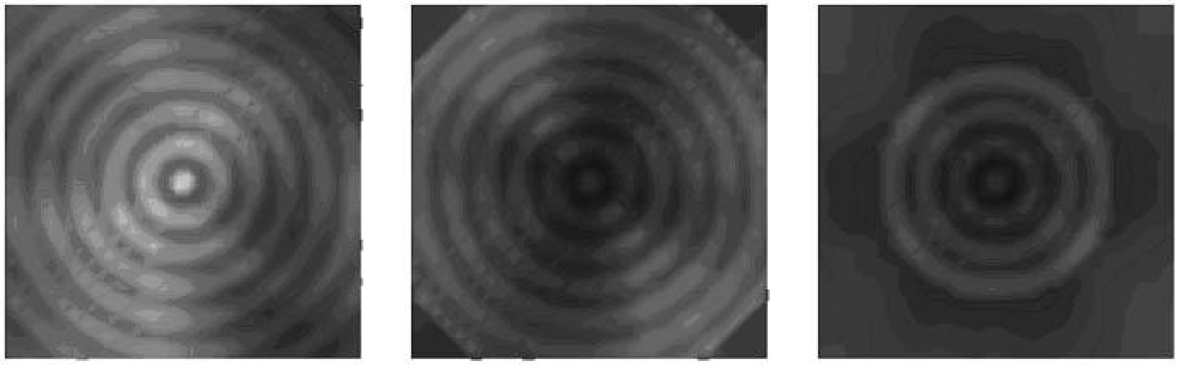
As we see from the above system, in order to find the field amplitude on the i^{th} particle, we must find the field on the *(i-1)* particle. To find a solution to Equation (5), the second assumption must be applied. Figures 8 – 10 show the results of numerical calculations on the system described above, using Bessel, Gaussian and Bessel – Gauss beams.

(a) (b) (c)

Figure 8: Passing BLB (a) with γ=50 mrad, λ=532nm, through a scattering medium which consist of 40 scattering particles with diameter 10µm, disposed in 4 layers; distance between layers is 0.5mm, distance l from ultimate layer to output plane is -1 mm (b); 6 mm (c).

(a) (b) (c)

Figure 9: Passing a Gaussian beam through the scattering medium; parameters are the same as in figure 8.



(a) (b) (c)

Figure 10: Passing a BGLB with γ=1.6 mrad, λ= 532nm, halfwidth w =100μm through scattering medium with parameters the same as in figure 8. l= 1 mm (a); 4 mm (b); 7mm (c).

6. CONCLUSION

In this paper we proposed a method of fast and accurate field calculations of BLB and BGLB propagating through media containing arbitrary obstacles. The method correctly predicts the self reconstruction properties of these beams, and can be applied to homogeneous or inhomogeneous media. The proposed method has shown excellent results in accuracy and calculation speed, and can be applied to any beam that has wave vectors lying on a cone.

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